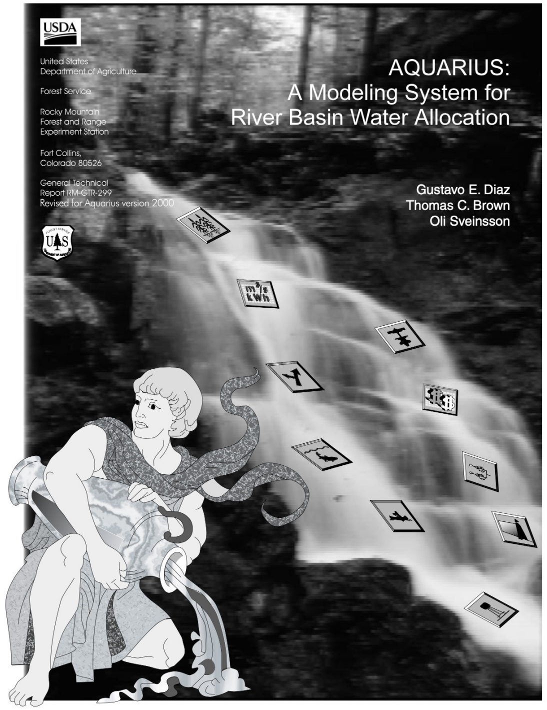
Water Resources Engineering and Management

(CIVIL-466, A.Y. 2024-2025)

5 ETCS, Master course



Prof. P. Perona
Platform of hydraulic constructions

Lecture 11-1: Quadratic Programming and AQUARIUS software

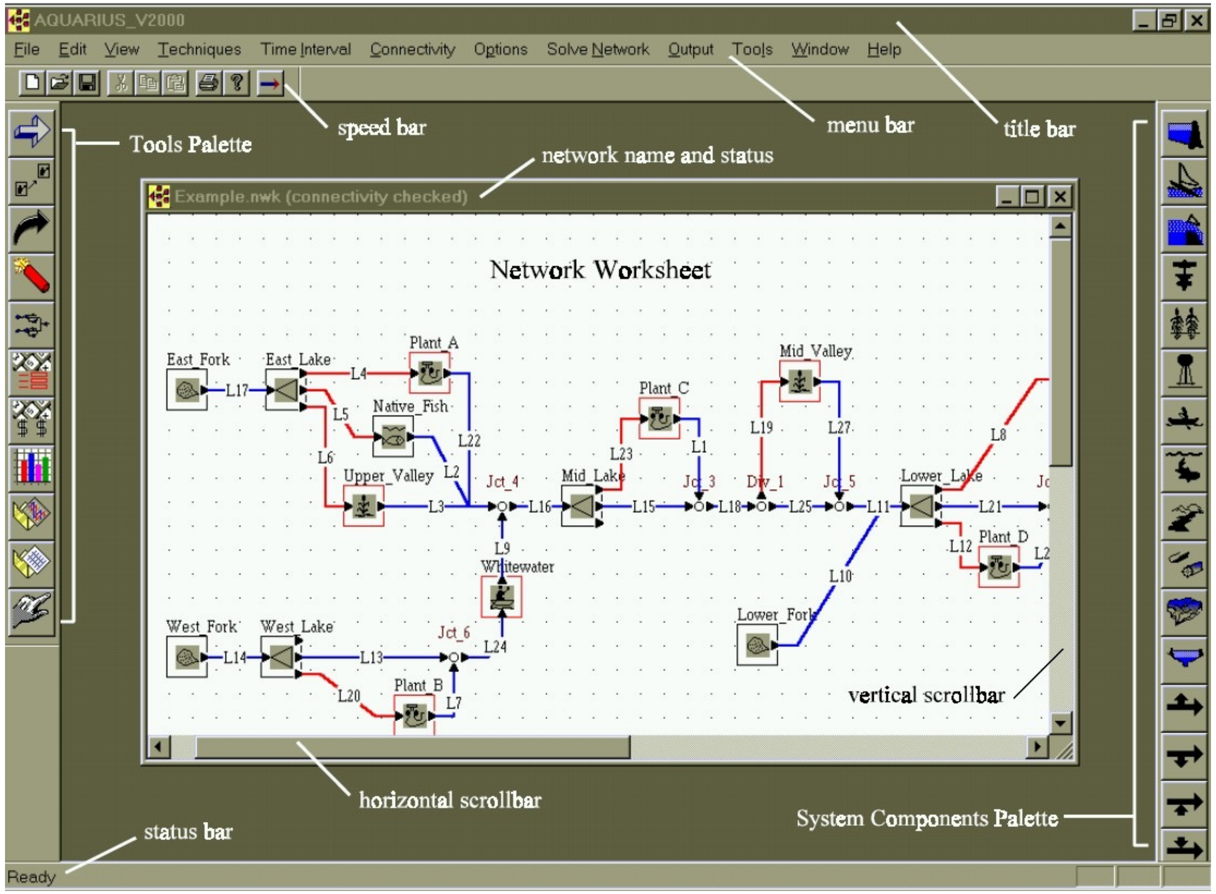
A modelling system for river basin water allocation (AQUARIUS)

PLATEFORME DE CONSTRUCTIONS HYDRAULIQUES

What is AQUARIUS?

AQUARIUS is a computer model devoted to the temporal and spatial allocation of flows among competing water uses in a river basin that can be used to evaluate either financial or economic efficiency.

AQUARIUS implements an object-oriented-programming language



Water resources models

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THREE TYPES:

- Simulation models. Are a conceptualization of a water system and are used to predict its hydrological response under predefined operational conditions

- Optimization models. Are able to take a decision (i.e., allocate water) depending on water inflow and demand. They automatically search for an optimal solution to the water allocation problem for all the time periods of interests, e.g. allow for dynamic programming.

- Network-flow models. Basically, these models can combine characteristics of both simulation and optimization but their performance are limited and does not allow for dynamic programming.

Which types of water management models?

Other examples:
REsources ALlocation Model (REALM)
Perera et al., J. Env. Manag. 2006

SWAM (Simplified water Allocation model)
https://hydrology.dnr.sc.gov/pdfs/swm/TechnicalReports/SWAMusermanv4dot0.pdf

Generalities on AQUARIUS

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Aquarius is a river basin model that integrates traditional and non traditional water uses using programming language and graphics capabilities (Object-Oriented-Programming)

What types of water uses does AQUARIUS consider?

- Traditional: flood control, hydropower, irrigation, urban water supply, etc.
- Non-traditional: preserving of biological geomorphological integrity of rivers, recreational activities, etc.

Type of value	Function (examples)	Type of function (examples)	Possible methods for economic evaluation
Utilitarian value	Commercial production	Hunting; fishing; plant harvesting	Monetary market value
	Protection	Shelter for man-made development; shelter for other habitats	Cost of flooding and erosion resulting from lack of protection; cost of replacement
	Pollution control; disposal of waste products; climate regulation	Absorption of pollution/sewage	Contingent valuation methods; unknown/indirect
User value	Quality of life	Providing natural beauty and a healthy and enjoyable environment	Hedonic house prices; contingent valuation methods
	Recreation	Bird watching; sport fishing; walking; jogging	Travel cost methods
	Education	School visits; projects	User willingness to pay
Nonuser value	Existence/bequest	Ecological role; existence for own sake	Incalculable; indirect
		Option for future use; bequest for future generations	Nonuser willingness to pay

Table 1. A schematic approach to economic evaluation of environmental services (adapted from Tunstall and Coker 1992).

Water uses imlemented in AQUARIUS

- Storage reservoir 
- Hydropower plants 
- Agricultural water use 
- Municipal and industrial water use 

Traditional

- Instream recreation water use 
- Reservoir recreation water use 
- Instream flow protection 

Non Traditional

What types of water uses does AQUARIUS include

Aquarius considers demand functions for some water uses (including related expenses for raw water)

Irrigation, Municipal, Commercial, Industrial, Hydropower, Riparian recreation,

For each water use the benefit function at a given instant of time *i* depends on how the related demand function can be allocated

Quadratic Programming: optimal water allocation

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1. Income distribution is accepted as given

2. Consumers are the best judges of their welfare (values assigned to goods and services are unquestioned)

3. Impacts outside the boundary of the analysis can be ignored

What are the main assumptions behind AQUARIUS's philosophy?

Benefits B and Costs C over *np* period of time are referred to the present value PV (*r* is the discount rate)

$$PV[r] = \sum_{i=0}^{np} \frac{(B_i - C_i)}{(1+r)^i}$$

(See also Lecture 6)

Target: maximise the total benefit function in discrete form

$$TB = \sum_{i=1}^{np} \sum_{j=1}^{nu} \int_0^{a_{ij}} f_{ij}(x_{ij})\, dx_{ij}$$

np = total number of time periods
nu = total number of water uses
a = level of consumption.

Benefit functions and solution techniques

Example: Irrigation demand area

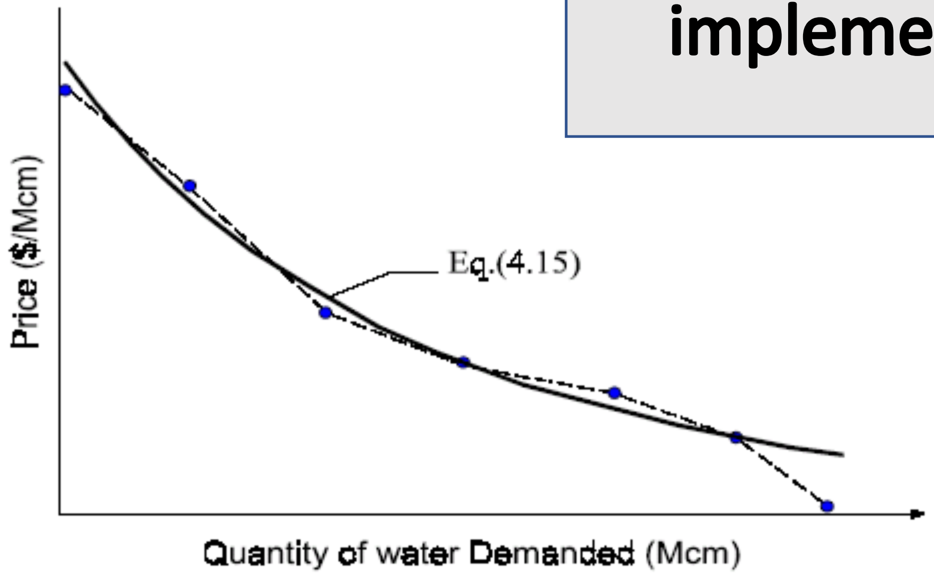
Let us assume that the marginal value of the demand function is

$$b^{IRR}\,[\$/Mcm] = a_3 \exp(-A/b_3)$$

A = total volume entering the IRR area

The integral of such a relation gives the total benefit at time *i*

$$B_i^{IRR} = \int_0^{A_i} a_{3_i} \exp(-z/b_{3_i})\,dz = a_{3_i} b_{3_i} [1 - \exp(-A_i/b_{3_i})]$$



How are marginal demand functions implemented?

Therefore the total benefit function for all the *np* period and all the *nu* water uses is

$$TB = \sum_{i=1}^{np} \sum_{j=1}^{nu} \int_0^{a_{ij}} f_{ij}(x_{ij})\, dx_{ij}$$

where $\int_0^{a_{ij}} f_{ij}(x_{ij})\, dx_{ij}$ is the benefit of the *j* water use at time *i*.

The overall objective is to maximize the total benefit function TB, i.e. matematically this can be written as

$$\underset{\mathbf{x}}{maximize}\ TB(\$) = \sum_{i=1}^{np} \sum_{u=1}^{nu} B_i^{\,u}$$

where the vector **x** is the set of the control variables.

What is the objective function to be maximized?

IMPORTANT NOTICE: Optimization is made among the water uses but also over time periods, which makes the software suitable for planning

Physical, operational and institutional constrictions such:

- reservoir storage limitations
- firm water supply
- seasonality of water supply
- firm energy production
- max/min instream flows
- max/min offstream diversion

Which physical and operational constraints are taken into account?

define the problem constraints (three types)

- equality constraints $\sum_{n=1}^{N} a_{kn} x_n = r_k$ *for* $k = 1, 2, \ldots,$.
- inequality constraints $\sum_{n=1}^{N} a_{kn} x_n \geq r_k$ *for* $k = K_{e+1}, \ldots, K$
- bounded variables $x_n^L \leq x_n \leq x_n^U$ *for some of the* x_n

Numerical solution method

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The water allocation problem involves a set of exponential/linear/constant demand functions and therefore determine a complex nonlinear objective function with multiple minima that may condition the maximization of TB.

Which solution method does AQUARIUS adopt?

Exception made for the nonlinear objective function, the nonlinear problem above is similar to a standard programming problem.

↓

Method of quadratic approximation

The objective function is reduced to a quadratic form with linear constraints

Quadratic approximations

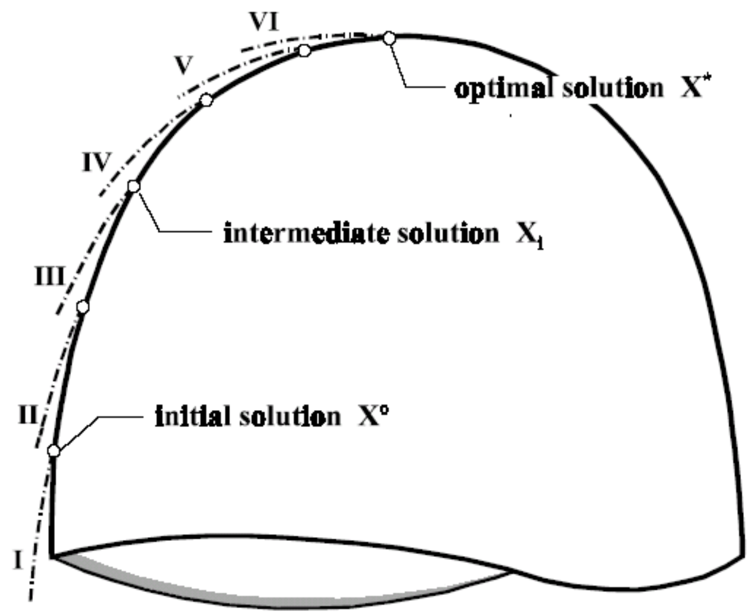
A quadratic approximation is a close representation of the nonlinear objective function $f(\mathbf{x})$

How do quadratic approximations work?

$f(\mathbf{x})$ can be expanded in Taylor series around a point $\mathbf{x}°$ which represents an Initial Feasible Solution (IFS)

$$f(\mathbf{x}) \approx f(\mathbf{x}^{\circ}) + \nabla f(\mathbf{x}^{\circ})^{T} \partial\mathbf{x} + \frac{1}{2} \partial\mathbf{x}^{T} \mathbf{H}(x^{\circ}) \partial\mathbf{x}$$

$$\underset{\mathbf{x}}{Max} \left[f(\mathbf{x}) = w + c^{T}\mathbf{x} + \frac{1}{2} \mathbf{x}^{T} \mathbf{Q} \mathbf{x} \right]$$



Standard Q.P. problem

- subject to the linear constraints $g(\mathbf{x}) = \mathbf{A}\mathbf{x} \geq \mathbf{r}$
- and nonnegativity conditions $\mathbf{x} \geq \mathbf{0}$

•NOTE: The gradient vector $\nabla f(\mathbf{x}°)$ and the Hessian $H(\mathbf{x}°)$ require, in principle, computation of first, second and second-cross partial derivatives which may introduce noise because of numerical calculation.

At which step is the numerical method implemented in AQUARIUS?

↓

The reason why the problem is approached in deterministic way is that derivatives are calculated algebraically and the assembling is therefore easier

The latter step is called **MATHEMATICAL CONNECTIVITY**

NOTICE: Building up the mathematical connectivity matrix is a milestone step for performing numerical optimization!

Aquarius then maximises the total return over a planning horizon, subject to all required operational restrictions

This is done iteratively using the set of variable obtained at each iteration as new starting point.

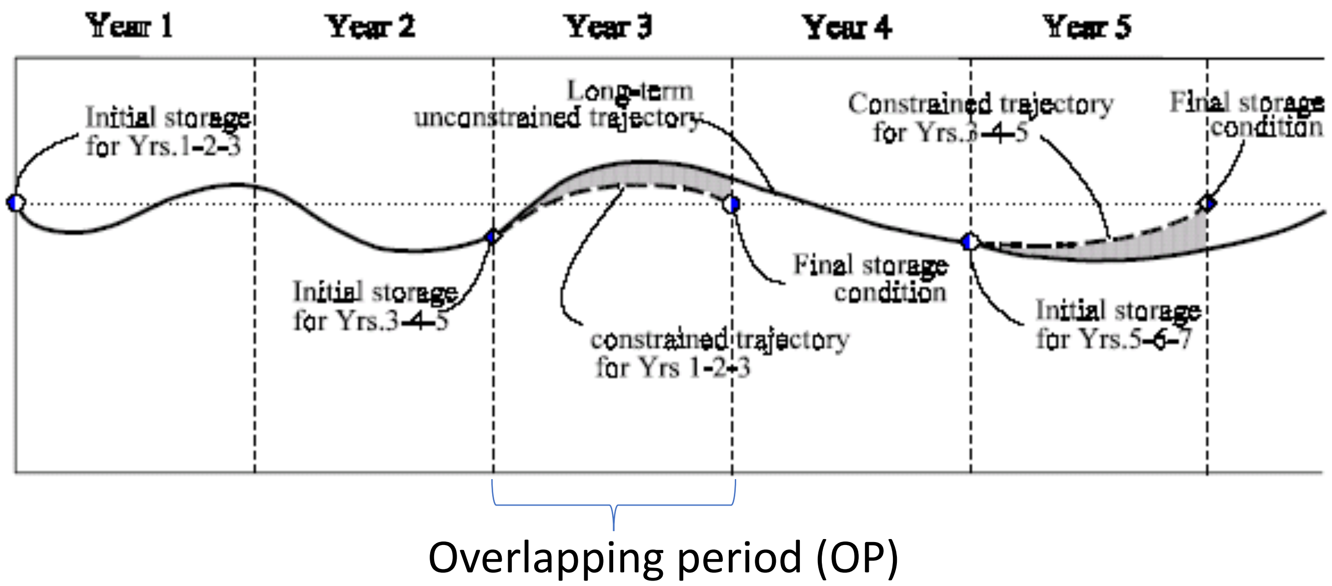
Optimal solution is reached when successive optimal values do not differ by more than the stipulated tolerance limit or when the maximum numer of iteration is reached

IMPORTANT: the optimal solution that is obtained by standard QP is only true for the approximated objective function defined by the Taylor expansion of the function $f(\mathbf{x})$.

How is numerical optimization achieved and when ?

Period of analysis and optimization horizon

Aquarius can be used in a *full optimization mode* for general planning purposes or in a *quasi-simulation mode* with restricted foresight capabilities, but efficient over very long time periods.



Period of analysis (PA): specify the length of the whole segment of time for which the model will simulate the allocation problem

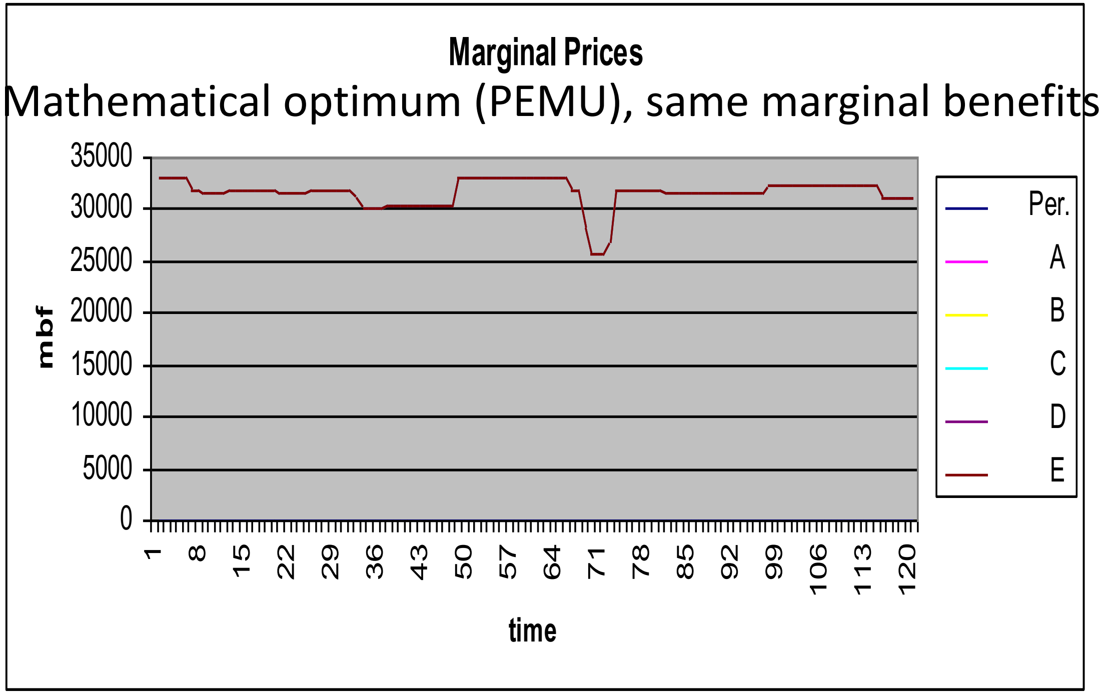
Optimization horizon (OH): specify how far in the future the model should look to built the optimal operational policies

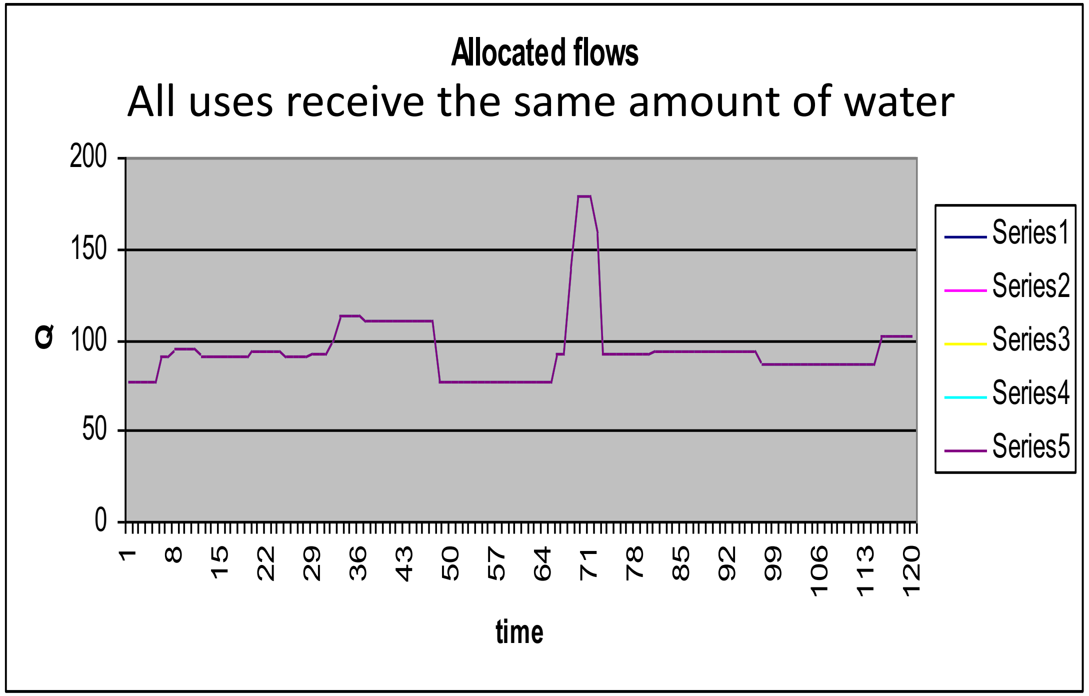
What are the period of analysis and the optimization horizon concepts?

Bounded vs unbounded solutions of optimal water allocation

Unbounded solutions

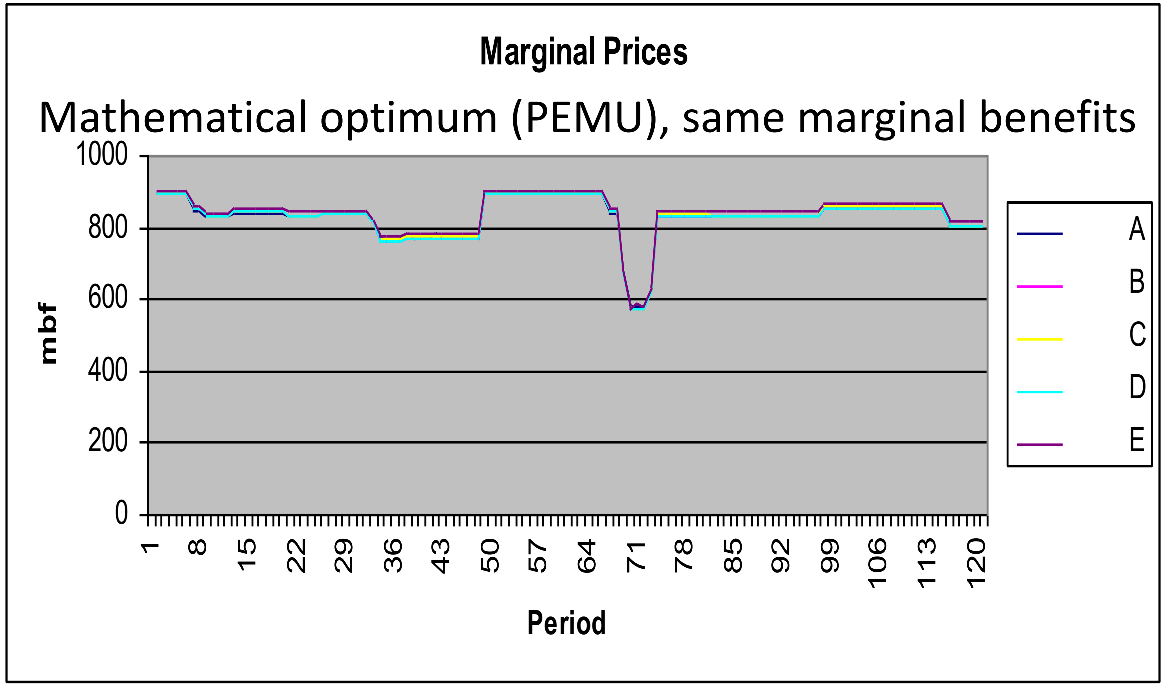
Case 1. Water users have in principle no constraints in the quantity of water that is being allocated, but equal marginal demand functions (i.e., exponential). What conclusions can you derive from the resulting solution in Figure below? Does the distribution of the marginal prices of the resource that is being allocated between water uses reflect your expectations?

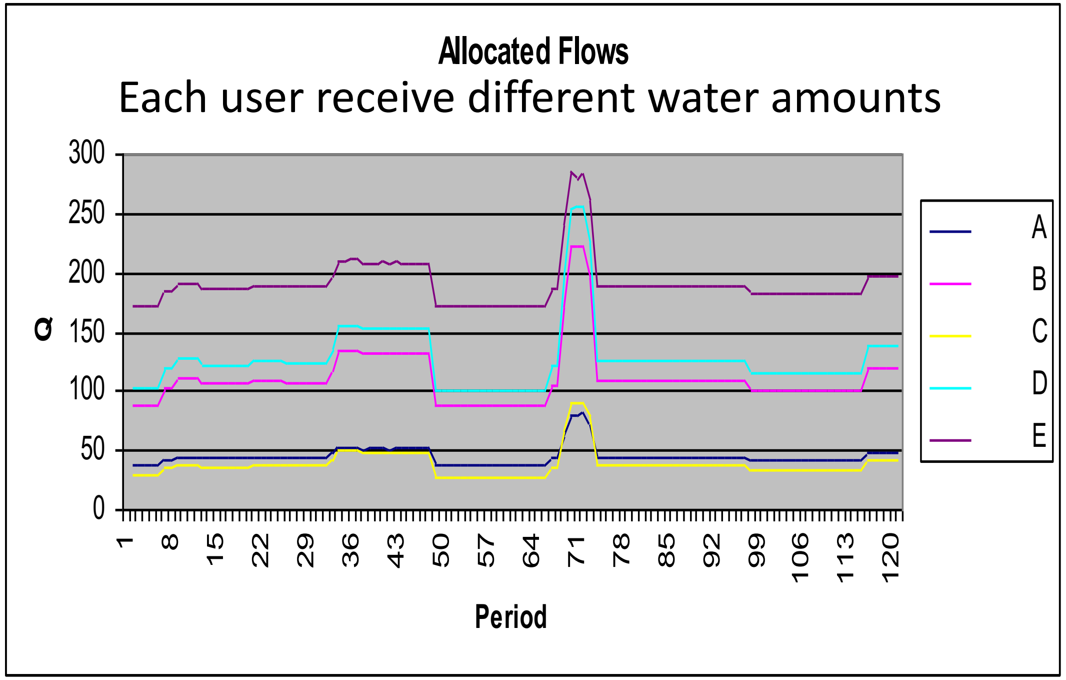




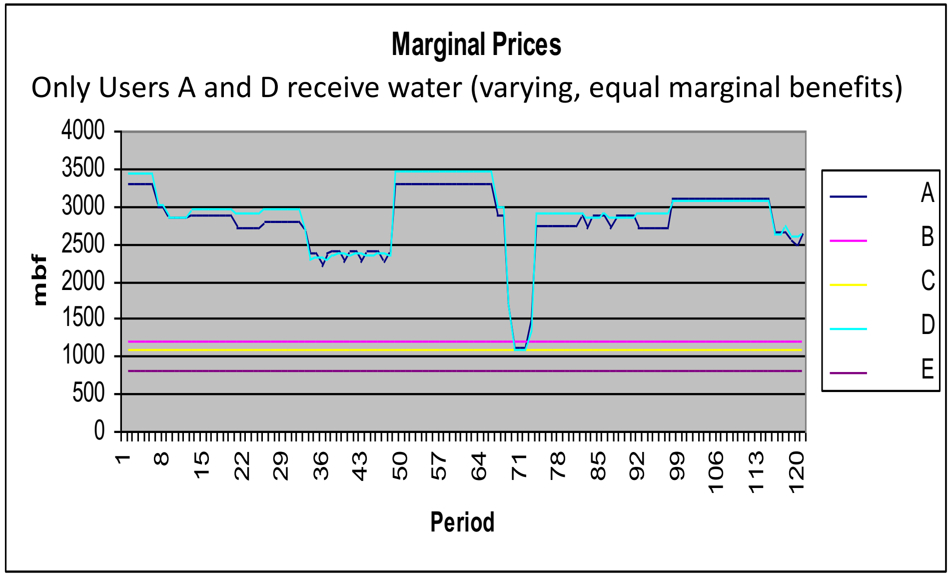
Unbounded solutions

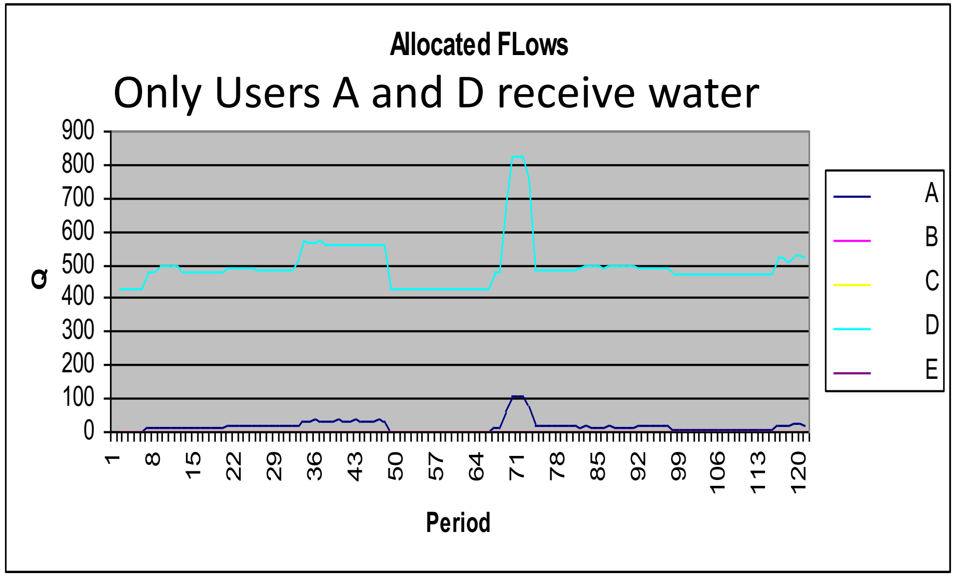
Case 2. Water users have in principle no constraints in the quantity of water that is being allocated, but they have different values for the parameters of the exponential demand function. Compared with Case 1, why does the optimisation allocate now a different amount of water among the users?



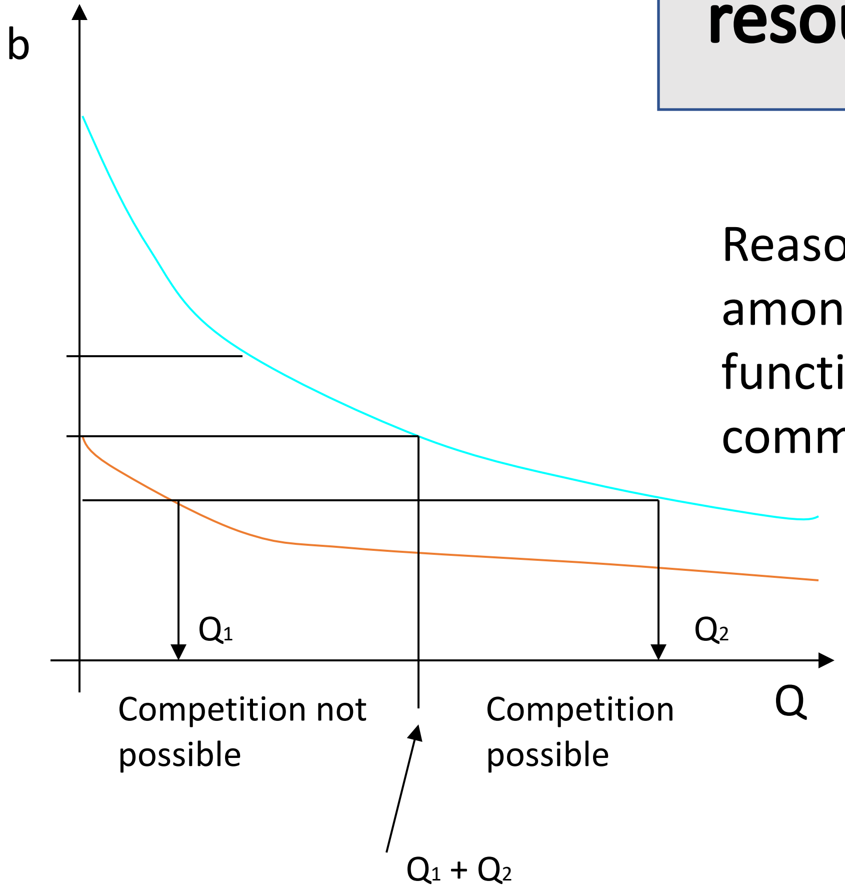


Resource-limited optimization (unbounded case)





Competition is not possible for Users B,C,E (too low demand function)

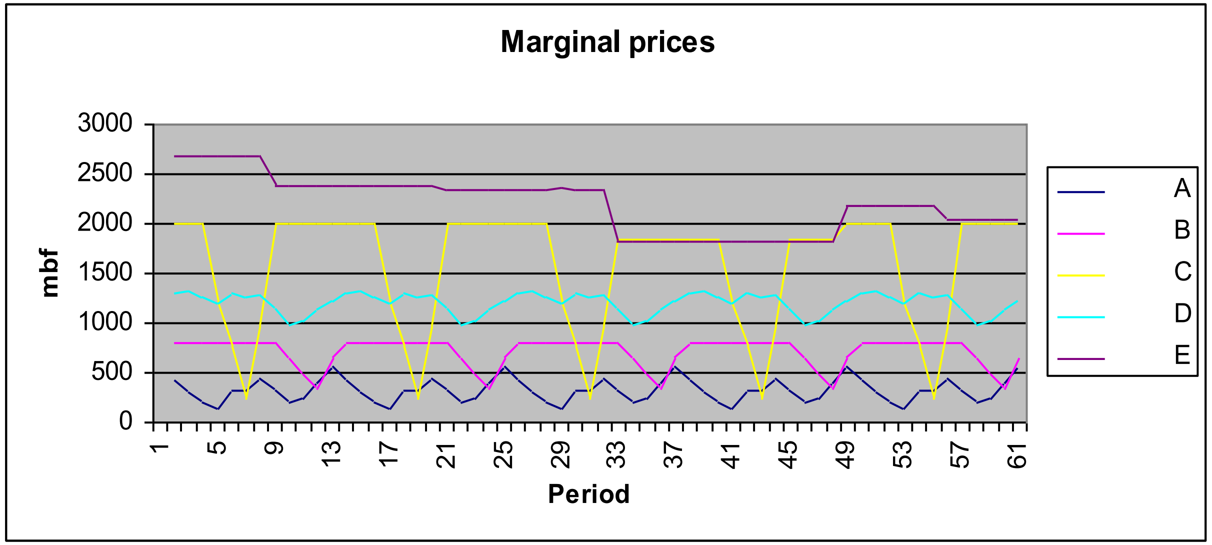


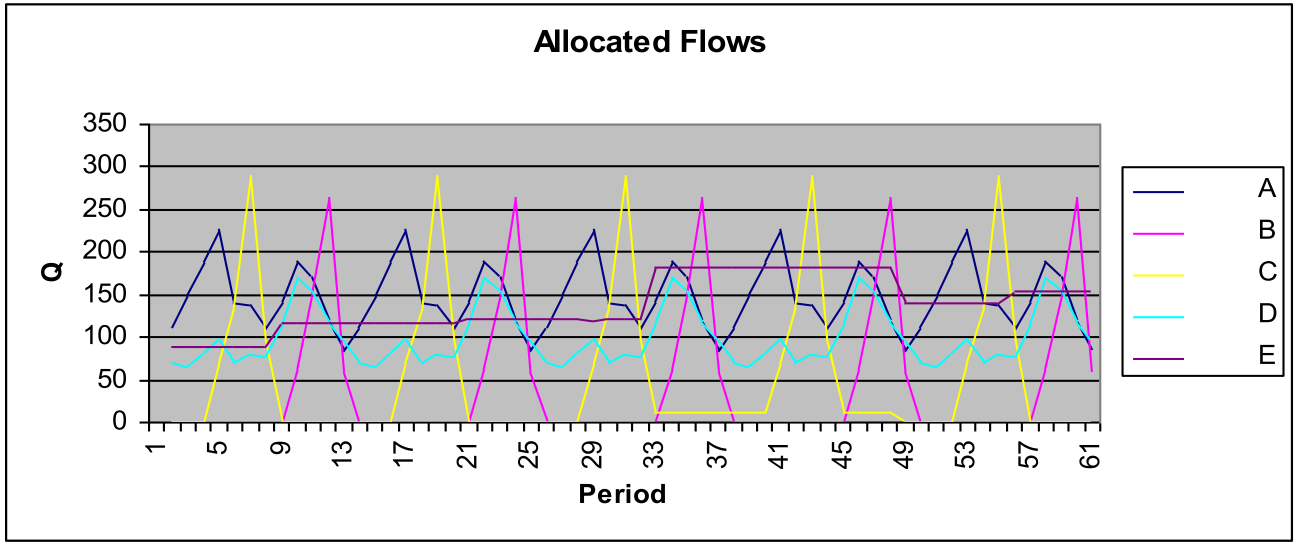
Which solution is found when the resource is limited?

Reason: Disproportion among marginal benefit functions (voice comments)

Bounded solutions (due to operational constraints)

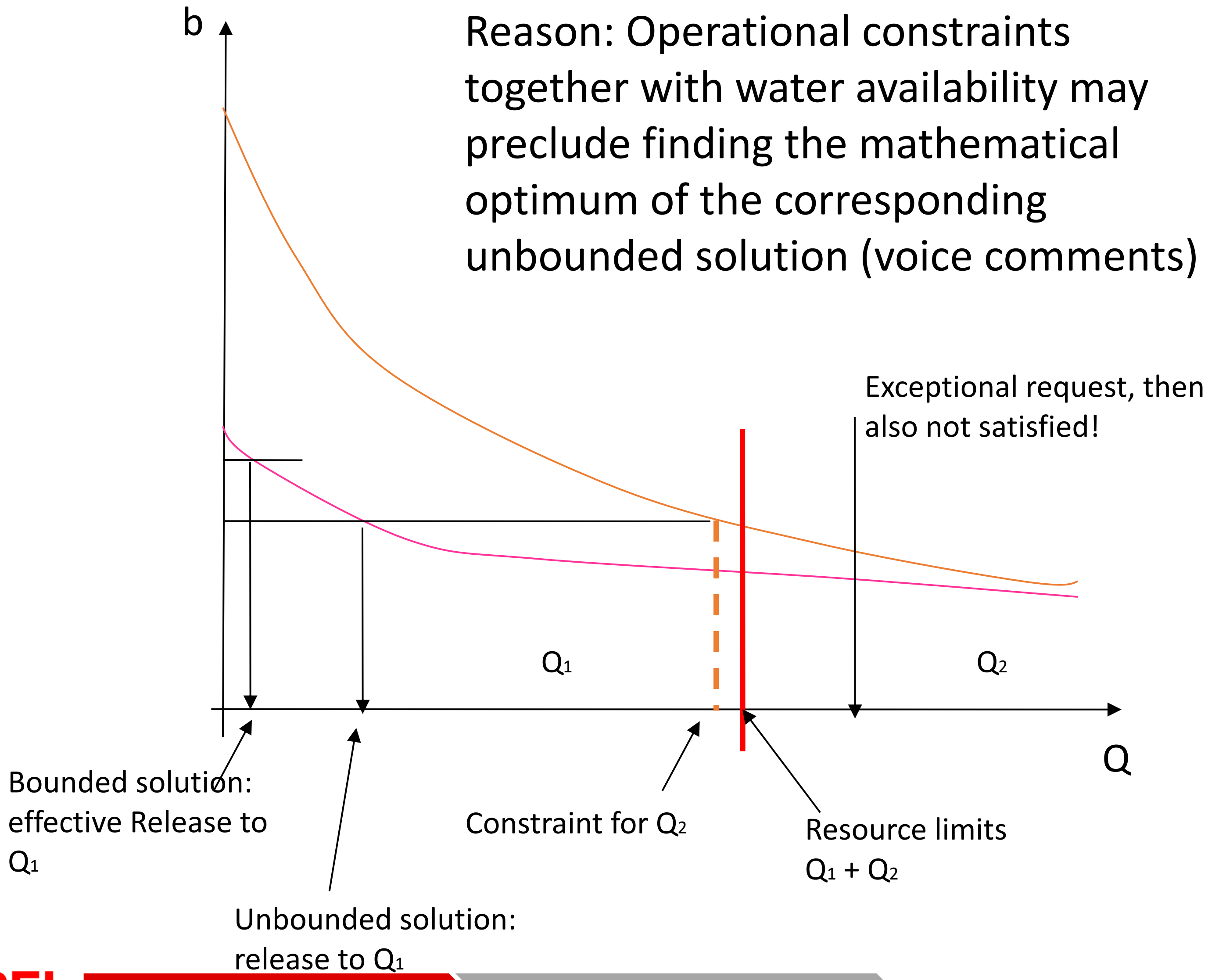
Case 3. All water uses have a constraint of minimum flow to be satisfied and either equal or different marginal benefit functions. The optimization over a period of N (free choice) years may return the results below.





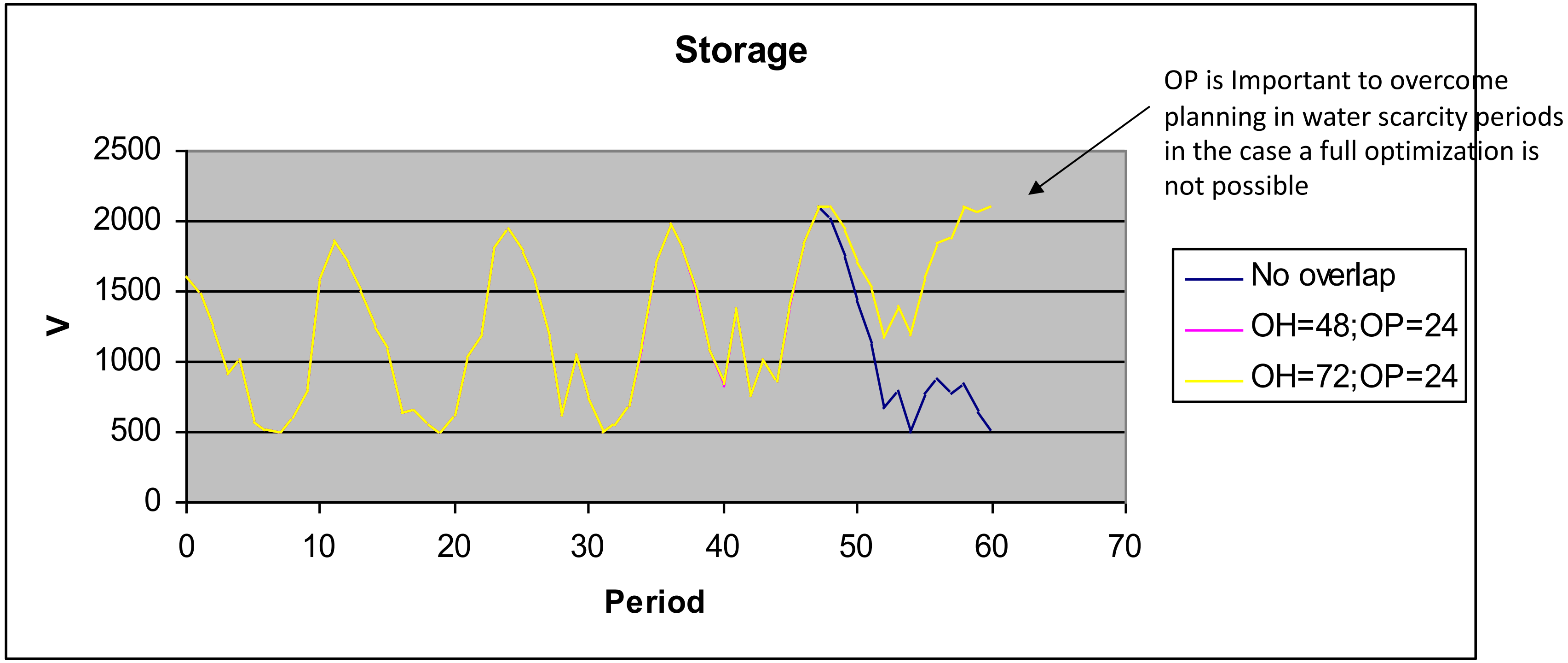
Which solution is found when the request is constrained?

Reason: Operational constraints together with water availability may preclude finding the mathematical optimum of the corresponding unbounded solution (voice comments)



Using Optimization horizon and overlapping periods

Case 4. If there is a long and unexpected drought at the beginning of a given year (e.g., here the 6th year) then best optimization can be found by playing with OH and OP parameters. The example below shows the case of a network whose best optimisation strategy must be performed over a total period of analysis of 120 Months. A substantial difference with OH=60 ; OP=0, and then with OH=72 ; OP=24. can be observed which is due to the full optimization capability of AQUARIUS



Software description and use

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EPFL

The river basin system

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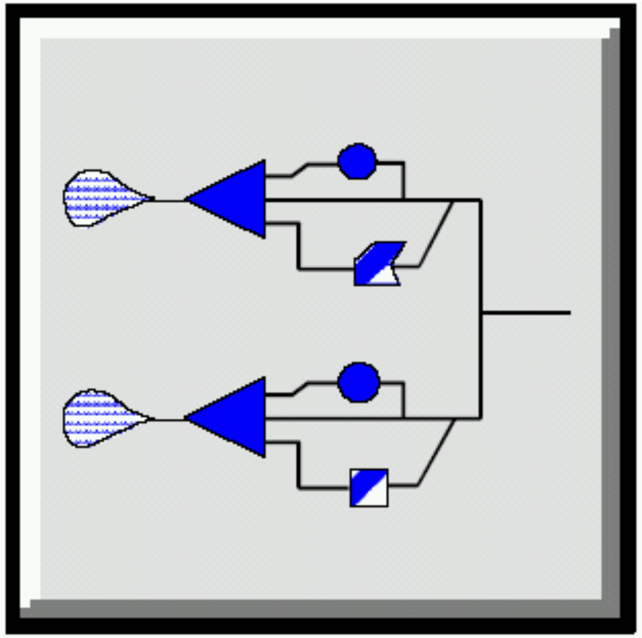
". . . a set of elements which interact in a regular manner. Every system must have a well defined boundary or rule which specifies and distinguishes that which is in the system from the environment in which it exists. There will be inputs and outputs across the boundary which must also be defined."

Hall & Dracup, 1970



System Components
(Interacting within the system)

- Natural basins
- Reservoirs
- Hydropower systems
- Urban water use
- Irrigation demand areas
- Instream recreation activities
- Etc.

System Boundaries (defined by the flow network)

- INPUTS (information)

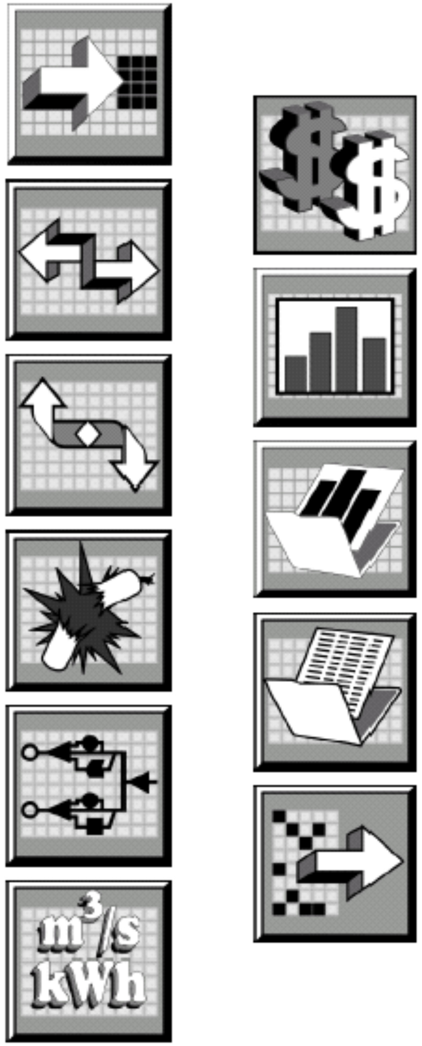
natural flows, water demand for irrigation, instream flow requirements, price for energy, etc.

- OUTPUTS(performance)

monetary values, energy produced, biological indexes, etc.

- Component Palette

Tools Palette



Flow Basin

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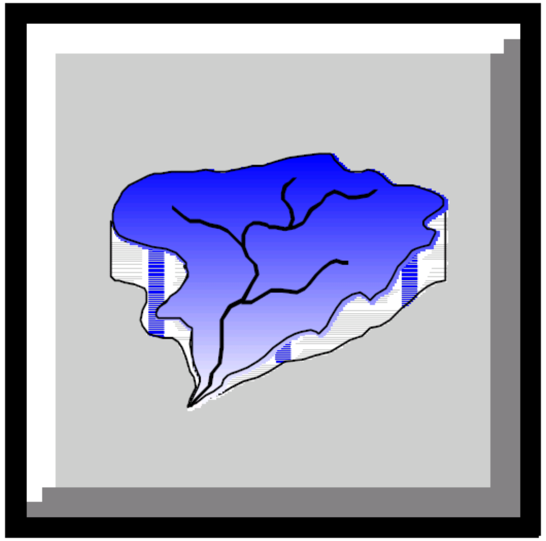

Flow Basin.. creates a water source area (basin) contributing uncontrolled flows.

Physical Characteristics

- Name of river/basin [alphanumeric]
- Input file name [Path….]

Requires specific inputs of inflows from a data file

Water uses

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- They require that both physical and operational characteristics are specified for the particular application.
- Moreover they are subjected to operational constraints which define the domain within the allocation problem must be optimized.
- Economic inputs need also to be declared in order to specify the demand functions (i.e., the marginal price that users are willing to pay)

EPFL

Storage reservoir

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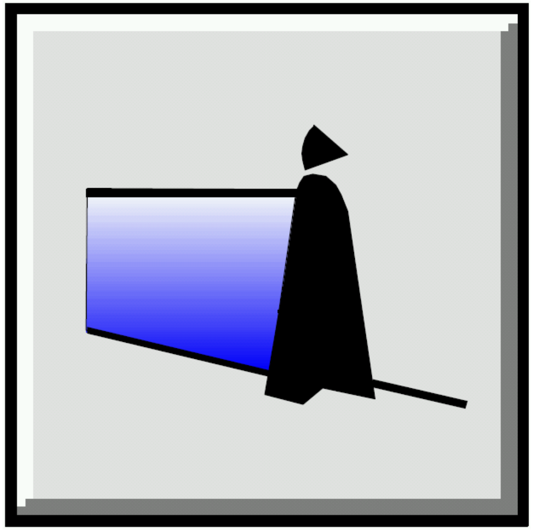
Transforms the random and periodic nature of flows into a series of releases that more closely corresponds with the seasonal water demand in a river basin



XI	=	controlled inflows
d^u	=	upstream decision variable
UI	=	uncontrolled inflows
NF^u	=	upstream natural flows
L^u	=	upstream reservoir spillage
XR	=	controlled releases
d	=	decision variable
UR	=	uncontrolled releases
L	=	reservoir spillage
E	=	reservoir evaporation

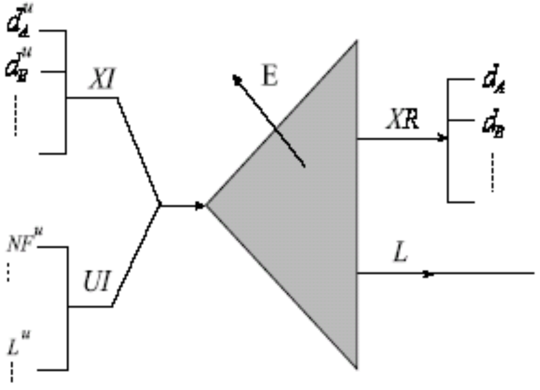


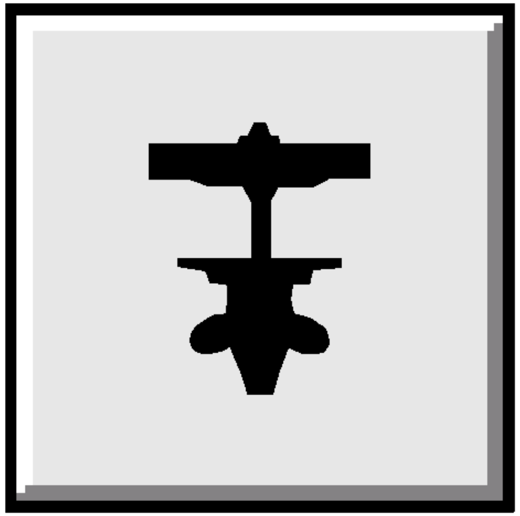
Figure 3.1 Variables used to describe storage dynamics in a reservoir.

<u>Requirements</u>: elevation-storage curve $H = c_1 S^{d_1}$

Minimum and maximum operating storage

Hydropower system

Hydropower.. creates a powerplant for hydroelectric generation.



Energy Generation Benefits
- Average energy price: P [$/MWh]
- Seasonal energy prices: P_s [$/MWh]
- Energy demand function; Parameters: a_2, b_2

Physical Characteristics
- Name of hydropower plant [alphanumeric]
- Installed capacity [MW]
- Design discharge [m^3/s]
- Turbine-generator efficiency [-]
- Energy rate vs. storage function, Parameters: a_1, b_1
- Tailwater Elevation vs. Storage function:

Firm Energy:
- Seasonal Pattern [GWh]

Maintenance Schedule
- Index of availability of the hydropower units [-]

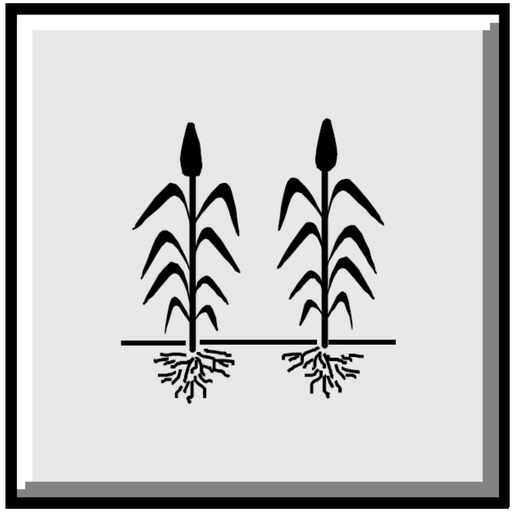
Maximum Release
- Seasonal values [Mcm]

Minimum Release
- Seasonal values [Mcm]

Operational Constraints (check-box)
- ☒ Minimum release
- ☒ Maximum release
- ☐ Firm energy

Agricultural water use

Irrigation.. creates an offstream demand area of agricultural water.



Physical Characteristics
- Name of irrigation area [alphanumeric]
- Coefficient of return flow [-]

Maximum Flow
- Seasonal values [Mcm]

Minimum Flow
- Seasonal values [Mcm]

Operational Constraints (check-box)
- ☐ Minimum flow
- ☐ Maximum flow
- ☐ Seasonal Pattern
- ☐ Annual firm water

Agricultural Benefits
- Seasonal demand function; Parameters: a_3, b_3

Municipal and industrial water use

Municipal and Industrial.. creates an offstream demand area for urban and industrial water.

Physical Characteristics
- Name of municipal and industrial area
- Coefficient of return flow

Maximum Flow
- Seasonal values [Mcm]

Minimum Flow
- Seasonal values [Mcm]

Operational Constraints (check-box)
- ☐ Minimum flow
- ☐ Maximum flow
- ☐ Seasonal Pattern
- ☐ Annual firm water

Urban and Industrial Water Supply Benefits
- Seasonal demand function; Parameters: a_4, b_4

Instream recreation water use

Instream Recreation.. creates a river reach with water based recreation activities.

Physical Characteristics

  Name of instream recreational area

Maximum Flow

  Seasonal values [Mcm]

Minimum Flow

  Seasonal values [Mcm]

Operational Constraints (check-box)

- ☐ Minimum flow
- ☐ Maximum flow

Instream Recreation Benefits

  Seasonal demand function; Parameters: a_5, b_5

Reservoir recreation water use

Reservoir Recreation.. creates a reservoir with water recreation activities.

Physical Characteristics
- Name of reservoir [alphanumeric]
- Elevation vs. storage function, Parameters: c_1, d_1
- Area vs. storage function: c_2, d_2

Evaporation
- Seasonal evaporation rates [mm]

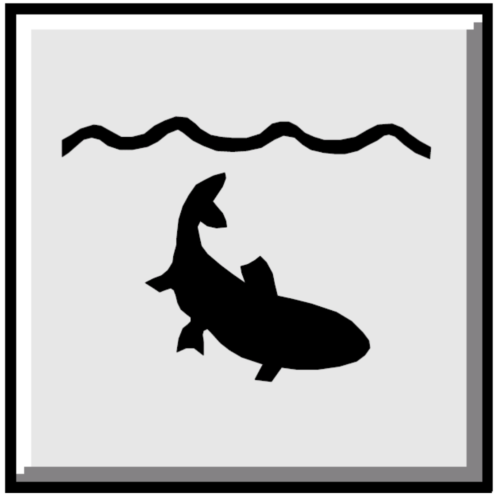
Operational Characteristics
- Initial storage [Mcm]
- Minimum storage [Mcm]
- Maximum storage [Mcm]
- Final storage [Mcm]

Operational Constraints (check-box)
- ☒ Minimum storage
- ☒ Maximum storage
- ☒ Final storage

Lake Recreation Benefits
- Seasonal demand function, Parameters: a_6, b_6, c_6

Instream flow protection

Instream Flow Protection.. creates a river reach designated for fish habitat protection.



Physical Characteristics

Name of conservation area [alphanumeric]

Maximum Flow

Seasonal values [Mcm]

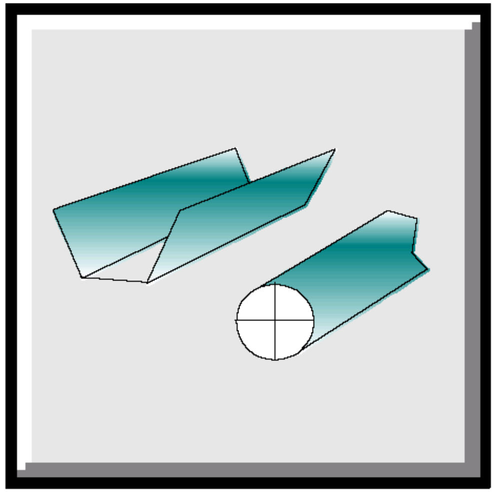
Minimum Flow

Seasonal values [Mcm]

Operational Constraints (check-box)

- ☐ Minimum flow
- ☐ Maximum flow

Flow conveyance structures



Canal/Pipeline.. creates a flow conveyance structure such as a canal or pipeline.

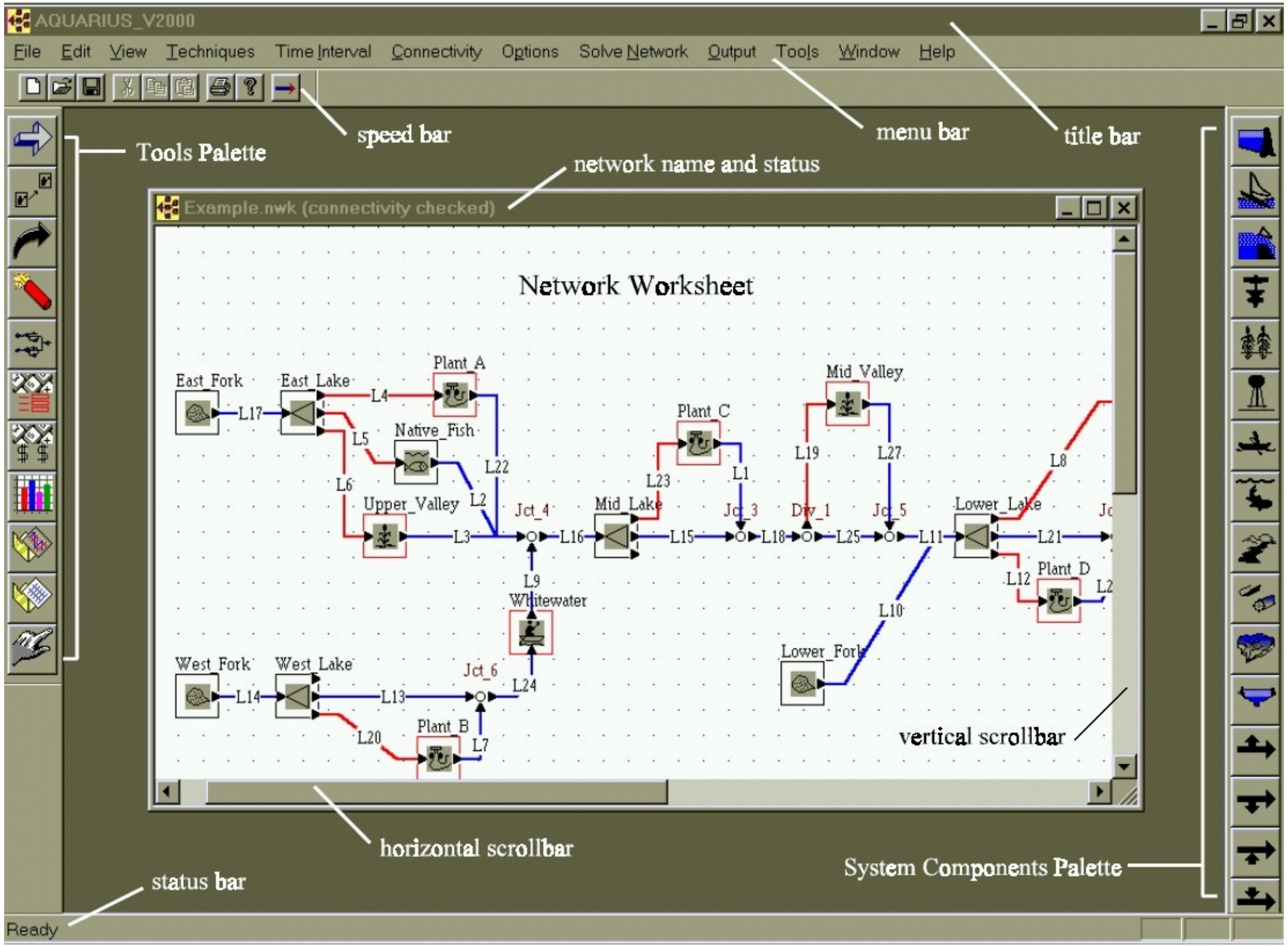
Physical Characteristics

- Name of river reach [alphanumeric]
- Description [alphanumeric]
- Length [m]
- Connectivity, From: __ To: __

Hydraulic Characteristics

- Maximum flow capacity [m^3/s]
- Flow velocity [m/s]
- Canal/pipe losses [l/km]

The model chart



Updating network connectivity

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Once the flow network is assembled, the model conducts a series of validation steps before proceeding to solve the water allocation problem.

- all WSCs are named

- all WSCs that compete for water have marginal prices different from zero

- Water uses such as IRA, IFP, FCA are instream users

- reservoirs have a waterway available for releasing uncontrolled spills

- Component connections are hydraulically sound

Table of Nodes for: Example.nwk

No.	Name	Type	Price Function	Connections Upstream	Connections DownStream
1	Big_City	MUNICIPAL	Yes	Lower_Lake	_
2	East_Fork	NATURALFLOW	No	_	East_Lake
3	Plant_D	POWERPLANT_VH	Yes	Lower_Lake	Jct_3

Table of Links for: Example.nwk

File

No.	Name	From	To	Description
1	L1	Plant_A	Jct_5	routing
2	L2	East_Fork	East_Lake	natflow
3	L3	Plant_C	Jct_4	routing
4	L4	Div_2	Mid_Valley	decision
5	L5	Upper_Valley	Jct_5	return
6	L6	Lower_Lake	Plant_D	decision
7	L7	Native_Fish	Jct_5	routing
8	L8	Lower_Lake	Big_City	decision
9	L9	East_Lake	Plant_A	decision

Optimization technique and time periods